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Nonlinear Coherent States and Some of Their Properties

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Abstract We construct nonlinear coherent states by the application of a deformed displacement operator acting upon the vacuum state and as approximate eigenstates of a deformed annihilation operator. These states are used to evaluate the temporal evolution of the average value of the momentum and the diplacement coordinate as well as their dispersions. We also construct even and odd combinations of these nonlinear coherent states and compute their second order correlation function in order to analyze their statistical behavior.

Keywords Nonlinear coherent states · Displacement operator · Deformed oscillator

1 Introduction

The coherent states for the electromagnetic field introduced by Glauber in 1963 [1–3] have played an important role in quantum optics. The development of lasers made it possible to prepare light fields which are very close to such states. Their behavior corresponds to that of a classical wave. Glauber showed that these states can be obtained from any one of three mathematical definitions: (i) as the right-hand eigenstates of the boson annihilation operator $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ with α a complex number, (ii) as those states obtained by the application of the displacement operator $D(\alpha)$ on the vacuum state of the harmonic oscillator $D(\alpha)|0\rangle = |\alpha\rangle$ with $D(\alpha) = \exp(\alpha a^{\dagger} - \alpha^{*} a)$, and (iii) as the quantum states with a minimum uncertainty relationship $(\Delta p)^2(\Delta q)^2 = 1/4$ with p and q the momentum and position operators. The same coherent states are obtained from the three mathematical definitions when one makes use of the harmonic oscillator algebra. For systems far from the ground state and for systems with a finite number of bound states the harmonic oscillator model is not adequate, therefore,

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there has arisen an interest to generalize these states to other systems which may possess different dynamical properties.

Nieto and Simmons [4–6] generalized the notion of coherent states to potentials other than the harmonic oscillator whose energy spectra have unequally spaced energy levels. The construction is such that the resultant states are localized, follow the classical motion and disperse as little as possible in time. The coherent states for a given potential $V(x)$ are defined as those which minimize the uncertainty relation equation. They were able to construct coherent states for the Poschl-Teller potential, the harmonic oscillator with a centripetal barrier and the Morse potential.

Gazeau and Klauder [7] proposed a generalization for systems with one degree of freedom possessing discrete and continuous spectra. These states also present continuity of labelling, a resolution of unity and temporal stability. The key point is the parametrization of the coherent states by two real values: an amplitude J, and a phase γ, instead of a complex value α.

Man'ko and collaborators [8] introduced coherent states of an f-deformed algebra as eigenstates of the annihilation operator $A = af(\hat{n})$ where $\hat{n} = a^\dagger a$ is the usual number operator and a, $a^\dagger$ are the annihilation and creation boson operators of the harmonic oscillator algebra respectively. These states present nonclassical properties such as squeezing and antibunching [9]. The properties of their even and odd combinations have also been studied by several authors [10–12].

More recently, the displacement operator method was generalized to the case of f-deformed oscillators, the main problem being that the commutator between the deformed operators A and $A^\dagger$ is not a number and the operator obtained by the replacement of the usual creation $a^\dagger$ and annihilation a, operators by their deformed counterparts can not be decomposed as a product. In order to circumvent this difficulty a *deformed* version of the displacement operator was introduced in [13, 14] with the disadvantage that it is nonunitary and does not displace the deformed operators in the usual form. Here we assume that the number operator function appearing in the commutator between the deformed operators can be replaced by a number. This allows us to apply the disentanglement theorem which yields a displacement operator which is approximately unitary and displaces the deformed annihilation A and creation $A^\dagger$ operators in the usual way.

2 Deformed Oscillator

Following Man'ko et al. [8] we introduce a deformed oscillator whose creation and annihilation operators are defined through

$$A = af(\hat{n}) = f(\hat{n}+1)a, \qquad A^\dagger = f(\hat{n})a^\dagger = a^\dagger f(\hat{n}+1), \quad \hat{n} = a^\dagger a,$$

with corresponding commutators

$$[\hat{n}, A] = -A, \qquad [\hat{n}, A^\dagger] = A^\dagger, \qquad [A, A^\dagger] = (\hat{n}+1)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n}).$$

It is reasonable to expect that the commutator between the deformed operators

$$[A, A^\dagger] = 1 + \hat{\phi} \tag{1}$$

be equal to the harmonic result plus a correction $\hat{\phi}$ which may be a function of the number operator and an anharmonicity parameter χ such that when $\chi \to 0$ we recover the harmonic result, i.e., $\hat{\phi}(\chi \to 0) = 0$. The deformation becomes fixed when one chooses the explicit form of the function $f(\hat{n})$, which in the harmonic case is simply $f(\hat{n}) = 1$.

A Hamiltonian with the form of a harmonic oscillator but written in terms of the deformed operators,

$$H_D = \frac{\hbar\Omega}{2}(A^\dagger A + AA^\dagger), \tag{2}$$

yields

$$H_D = \frac{\hbar\Omega}{2}(\hat{n} f^2(\hat{n}) + (\hat{n}+1) f^2(\hat{n}+1)).$$

We now choose the deformation function

$$f^2(\hat{n}) = \frac{1-\chi\hat{n}}{1-\chi}, \tag{3}$$

where $\chi = 1/(2N+1)$ with N an integer. Then Hamiltonian becomes

$$H_D = \hbar\Omega' \left(\hat{n} + 1/2 - \chi\left(\hat{n} + \frac{1}{2}\right)^2 - \frac{\chi}{4}\right), \tag{4}$$

with $\Omega' = \Omega/(1-\chi)$. Equation (4) is the Hamiltonian of a harmonic oscillator plus a non-linear contribution quadratic in the number operator. Thus, its eigenfunctions $|n\rangle$ coincide with those of the harmonic oscillator, but it supports only a finite number of bound states $N+1$; n can take the values $0, 1, \ldots, N$. Apart from a constant term, the resulting spectra is similar to that of the Morse and the Pöschl-Teller Hamiltonians [15],

$$E_M = \omega_e\left(n + \frac{1}{2}\right) - \chi_e\omega_e\left(n + \frac{1}{2}\right)^2. \tag{5}$$

When we substitute (3) into the commutation relations for operators A, $A^\dagger$ and $\hat{n}$ we get

$$[A, A^\dagger] = 1 - \frac{2\chi\hat{n}}{1-\chi}. \tag{6}$$

As mentioned before, coherent states for the harmonic oscillator, or *field coherent states* can be generated from three alternative definitions. For the case of general potentials several possibilities have been explored. However, the results depend in general on the kind of generalization employed [4, 7, 11, 14, 16, 17].

3 Displacement Operator and Coherent States

When using the displacement operator method to generate the coherent states for deformed potentials, one faces the problem that the commutator between the deformed operators A and $A^\dagger$ is not a number, as a consequence, the displacement operator $D(\alpha) = \exp[-\alpha^* A + \alpha A^\dagger]$ obtained by the replacement of the usual operators a, $a^\dagger$ by their deformed counterparts can not be written in a product form. In [13] the problem was circumvented by the interchange of $A^\dagger$ by $A_f^\dagger = 1/f(\hat{n})a^\dagger = (1/f(\hat{n})^2)A^\dagger$ which yields the deformed displacement operator

$$D_f(\alpha) \equiv \exp[-\alpha^* A + \alpha A_f^\dagger] = e^{-|\alpha|^2/2} e^{\alpha A_f^\dagger} e^{-\alpha^* A}.$$

The operator $D_f(\alpha)$ displaces A by α and $D_{1/f}(\alpha)$ displaces $A^\dagger$ by α^*. However, $A_f^\dagger$ is not the adjoint of A and the deformed displacement operator is not unitary. A similar method was used by Roy et al. [10, 18, 19] in their study of nonlinear coherent states and their nonclassical properties.

In this work we propose a different approach to avoid the difficulties due to the fact that the commutator between the deformed operators is a function of the number operator. To that end we *assume* that the number operator appearing in the commutator between A and $A^\dagger$ (see (6)) can be replaced by a function of the average occupation number $n_\alpha = \langle\alpha|\hat{n}|\alpha\rangle$. Then, using the commutation relation $[A, A^\dagger] \simeq 1 + \phi$, where ϕ is obtained from $\hat{\phi}$ by replacing the operator $\hat{n}$ by the number n_α, we define an approximate displacement operator in normal order as

$$D_D(\alpha) \equiv e^{\alpha A^\dagger} e^{-\alpha^* A} e^{-\frac{1}{2}|\alpha|^2(1+\phi)}. \tag{7}$$

Notice however that the deformed operators A, $A^\dagger$ in $D_D(\alpha)$ contain the number *operator* function $f(\hat{n})$. Within the same approximation, the inverse of $D_D(\alpha)$ is

$$D_D(\alpha)^{-1} = e^{\frac{1}{2}|\alpha|^2(1+\phi)} e^{\alpha^* A} e^{-\alpha A^\dagger}. \tag{8}$$

From (7) it is also clear that $D_D(\alpha)^\dagger = D_D(-\alpha)$. In order to see whether or not the operator $D_D(\alpha)$ is approximately unitary we calculate

$$D_D(\alpha)^\dagger = e^{-\alpha A^\dagger} e^{\alpha^* A} e^{-\frac{1}{2}|\alpha|^2(1+\phi)}. \tag{9}$$

Multiplying on the left by $e^{\alpha^* A} e^{-\alpha^* A}$ we obtain

$$D_D(\alpha)^\dagger = e^{\alpha^* A} e^{-\alpha^* A} e^{-\alpha A^\dagger} e^{\alpha^* A} e^{-\frac{1}{2}|\alpha|^2(1+\phi)}. \tag{10}$$

Now, we approximate the product

$$e^{-\alpha^* A} e^{-\alpha A^\dagger} e^{\alpha^* A} \simeq e^{|\alpha|^2(1+\phi)} e^{-\alpha A^\dagger}, \tag{11}$$

where we have used the commutators

$$[A, A^\dagger] \simeq 1 + \phi, \qquad [A, A^{\dagger^n}] \simeq n A^{\dagger^{n-1}}(1+\phi), \qquad [A, e^{-\alpha A^\dagger}] \simeq -\alpha(1+\phi)e^{-\alpha A^\dagger}.$$

Upon substitution we obtain,

$$D_D(\alpha)^\dagger \simeq e^{\alpha^* A} e^{|\alpha|^2(1+\phi)} e^{-\alpha A^\dagger} e^{-\frac{1}{2}|\alpha|^2(1+\phi)} = D_D(\alpha)^{-1}, \tag{12}$$

which shows that the displacement operator $D_D(\alpha)$ is approximately unitary.

With the operator $D_D(\alpha)$ we may displace both A and $A^\dagger$

$$D_D(\alpha)^\dagger A D_D(\alpha) \simeq A + \alpha(1+\phi), \tag{13}$$

$$D_D(\alpha)^\dagger A^\dagger D_D(\alpha) \simeq A^\dagger + \alpha^*(1+\phi). \tag{14}$$

We must now analyze under what conditions the approximations leading to (7) are applicable. Following [20] we define $F(\lambda)$ through

$$\exp[\lambda(-\alpha^* A + \alpha A^\dagger)] = \exp[\lambda(\alpha A^\dagger)]F(\lambda).$$

Differentiation with respect to the scalar parameter λ and a little algebra lead to

$$\frac{dF(\lambda)}{F(\lambda)} = -\alpha^* e^{-\lambda\alpha A^\dagger} A e^{\lambda\alpha A^\dagger} d\lambda. \tag{15}$$

The transformation in the right hand side is

$$e^{-\lambda\alpha A^\dagger} A e^{\lambda\alpha A^\dagger} = A + \lambda\alpha(1+\hat{\phi}) + \frac{1}{2}\lambda^2\alpha^2[A^\dagger, \hat{\phi}] + \cdots. \tag{16}$$

From (6), we identify

$$\hat{\phi} = C\hat{n}, \tag{17}$$

where $C = -2\chi/(1-\chi)$, so that the commutator

$$[A^\dagger, \hat{\phi}] = -CA^\dagger$$

and the leading term in (16) becomes

$$e^{-\lambda\alpha A^\dagger} A e^{\lambda\alpha A^\dagger} \simeq A + \lambda\alpha(1 + C\hat{n}) - \frac{1}{2}C\lambda^2\alpha^2 A^\dagger. \tag{18}$$

Substitution into (15) and integration leads to

$$F(\lambda) = \exp\left[-\alpha^*\left(A\lambda + \frac{1}{2}\alpha(1+C\hat{n})\lambda^2 - \frac{1}{6}C\alpha^2 A^\dagger\lambda^3\right)\right].$$

Taking $\lambda \to 1$ we obtain finally

$$\exp[-\alpha^* A + \alpha A^\dagger] \simeq \exp[\alpha A^\dagger]\exp\left[-\alpha^*\left(A + \frac{1}{2}\alpha(1+C\hat{n}) - \frac{1}{6}C\alpha^2 A^\dagger\right)\right]. \tag{19}$$

Proceeding in a similar form with the second exponential in the right hand side of (19) we get

$$e^{-\alpha^*(A+\frac{1}{2}\alpha(1+C\hat{n})-\frac{1}{6}C\alpha^2A^\dagger)} \simeq e^{-\alpha^* A} e^{-\frac{|\alpha|^2}{2}(1+C\frac{|\alpha|^2}{6})} e^{-C\frac{|\alpha|^2}{2}(\hat{n}+\frac{\alpha}{3}A^\dagger-\frac{\alpha^*}{2}A)}.$$

Then,

$$\exp[-\alpha^* A + \alpha A^\dagger] \simeq \exp[\alpha A^\dagger]\exp[-\alpha^* A]\exp\left[-\frac{|\alpha|^2}{2}(1+C\hat{n})\right]. \tag{20}$$

This expression is valid whenever $|\alpha^2 C| \ll 1$. As typically $\chi \ll 1$, the condition for the validity of (20) is $|\alpha|^2\chi \ll 1$. This imposes an upper bound on the value of α. Since the average energy of the oscillator is a function of the *size* α of the coherent state, for a Morse-like coherent state there must exist a maximum value of $|\alpha|$ in order to remain in the bound part of the spectrum [22].

A normalized, nonlinear, approximately coherent state is obtained by application of the approximate displacement operator $D_D(\alpha)$ upon the vacuum state,

$$|\alpha\rangle_D \equiv \mathcal{N} e^{-\frac{|\alpha|^2}{2}(1+\phi)} e^{\alpha A^\dagger} e^{-\alpha^* A}|0\rangle = \mathcal{N} e^{-\frac{1}{2}|\alpha|^2(1+\phi)} \sum_{l=0}^{\infty} \frac{\alpha^l}{\sqrt{l!}} f(l)!|l\rangle \tag{21}$$

where $f(l)! = f(1)f(2)\cdots f(l)$ and $\mathcal{N}$ is a normalization constant. Imposing the condition $_D\langle\alpha|\alpha\rangle_D = 1$, we obtain

$$|\alpha\rangle_D = \left[\sum_{k=0}^{\infty}\frac{|\alpha|^{2k}}{k!}[f(k)!]^2\right]^{-1/2}\sum_{l=0}^{\infty}\frac{\alpha^l}{\sqrt{l!}}f(l)!|l\rangle = N_D\sum_{l=0}^{\infty}\frac{\alpha^l}{\sqrt{l!}}f(l)!|l\rangle, \qquad (22)$$

where

$$N_D = \left[\sum_{k=0}^{\infty}\frac{|\alpha|^{2k}}{k!}[f(k)!]^2\right]^{-1/2}$$

is a normalization constant which is a function of the parameter α. Notice that the function ϕ does not appear in the normalized approximate coherent state. The expression (22) we obtained for the state $|\alpha\rangle_D$ is identical to that obtained in [18, 21] with the use of a *deformed* displacement operator ((13) of [18]). The state $|\alpha\rangle_D$ has a nonzero projection on every Fock state $|n\rangle$

$$\langle n|\alpha\rangle_D = N_D\frac{\alpha^n}{\sqrt{n!}}f(n)!. \qquad (23)$$

The probability that n excitations will be found in the state $|\alpha\rangle_D$ is given by

$$p(n) = |\langle n|\alpha\rangle_D|^2 = |N_D|^2\frac{|\alpha|^{2n}}{n!}|f(n)!|^2 \qquad (24)$$

which approaches a Poisson distribution in n with parameter $|\alpha|^2$ in the limit of small χ.

4 Average Values

The normalized coherent states defined as eigenstates of the deformed annihilation operator A are given by [8, 14]:

$$|\alpha\rangle_A = N_f\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}f(n)!}|n\rangle \qquad (25)$$

where N_f is a normalization constant and differ from those obtained by application of the displacement operator (22) mainly in the fact that the factorial $f(n)!$ appears in the numerator in one case and in the denominator in the other. It was shown in [22] that the average value of the position and momentum operators pertinent to a Morse-like potential taken between the coherent states given in (25) present a conduct that resembles very closely the classical one. In this section we evaluate the averages of position and momentum for a Morse-like potential between the coherent states obtained via the displacement operator and present a comparison between them.

For a Morse-like potential, the coordinate and momentum may be written as polynomials in terms of deformed operators [23]

$$x_D \simeq \sqrt{\frac{\hbar}{2m\Omega'}}\left(f_{00} + f_{10}A^\dagger + Af_{01} + f_{20}A^{\dagger 2} + A^2 f_{02}\right), \qquad (26)$$

$$p_D \simeq i\sqrt{\frac{\hbar m\Omega'}{2}}\left(g_{10}A^\dagger + Ag_{01} + g_{20}A^{\dagger 2} + A^2 g_{02}\right) \qquad (27)$$

where the expansion coefficients f_{ij} and g_{ij} are known functions of the number operator [22]. Their temporal evolution is calculated taking the averages between coherent states $|\alpha(t)\rangle_D = U(t,t_0)|\alpha(t_0)\rangle_D$ with $U(t,t_0)$ the time evolution operator, t_0 the initial time. In this work we will restrict ourselves to the bounded part of the spectrum, so that employing the Hamiltonian (4) we obtain

$$
{}_D\langle\alpha(t)|O|\alpha(t)\rangle_D \simeq |N_D|^2 \sum_{n=0}^{N}\sum_{m=0}^{N} \frac{\alpha^{*^m}\alpha^n f(m)!f(n)!}{\sqrt{m!}\sqrt{n!}} \times e^{i\Omega'(t-t_0)(m-n)(1-\chi-(m+n)\chi)}\langle m|O|n\rangle \tag{28}
$$

where O represents either x_D or p_D.

For the usual coherent states the probability of occupying the n-th level is given by a Poissonian function

$$
P(n) = \frac{|\alpha|^{2n}}{n!}e^{-|\alpha|^2} \tag{29}
$$

with mean excitation number $\langle\alpha|\hat{n}|\alpha\rangle = |\alpha|^2$ and dispersion

$$
\sigma_n = \langle\alpha|\hat{n}^2|\alpha\rangle - \langle\alpha|\hat{n}|\alpha\rangle^2 = |\alpha|^2
$$

so that the ratio $\sigma_n/\langle\alpha|\hat{n}|\alpha\rangle = 1$. For the nonlinear coherent states obtained from the deformed displacement operator introduced in this work and as eigenstates of the deformed annihilation operator we get respectively

$$
{}_D\langle\alpha|\hat{n}|\alpha\rangle_D \simeq |N_D|^2 \sum_{n=0}^{N} \frac{|\alpha|^{2n}[f(n)!]^2}{(n-1)!}, \qquad {}_A\langle\alpha|\hat{n}|\alpha\rangle_A \simeq |N_f|^2 \sum_{n=0}^{N} \frac{|\alpha|^{2n}}{[f(n)!]^2(n-1)!}
$$

and

$$
{}_D\langle\alpha|\hat{n}^2|\alpha\rangle_D \simeq |N_D|^2 \sum_{n=0}^{N} \frac{n|\alpha|^{2n}[f(n)!]^2}{(n-1)!}, \qquad {}_A\langle\alpha|\hat{n}^2|\alpha\rangle_A \simeq |N_f|^2 \sum_{n=0}^{N} \frac{n|\alpha|^{2n}}{[f(n)!]^2(n-1)!}
$$

so that the ratios

$$
g_D^{(2)} = \frac{{}_D\langle\alpha|\hat{n}^2|\alpha\rangle_D - {}_D\langle\alpha|\hat{n}|\alpha\rangle_D^2}{{}_D\langle\alpha|\hat{n}|\alpha\rangle_D}, \tag{30}
$$

$$
g_A^{(2)} = \frac{{}_A\langle\alpha|\hat{n}^2|\alpha\rangle_A - {}_A\langle\alpha|\hat{n}|\alpha\rangle_A^2}{{}_A\langle\alpha|\hat{n}|\alpha\rangle_A} \tag{31}
$$

will be equal, less than or grater than unity depending upon the size of the coherent state and the deformation function.

5 Numerical Results

In Fig. 1, we see that the deformed coherent states $|\alpha\rangle_A$ (dotted line) present Poissonian behaviour for a small region around $\alpha = 0$ and sub-Poissonian (anti bunching) behaviour

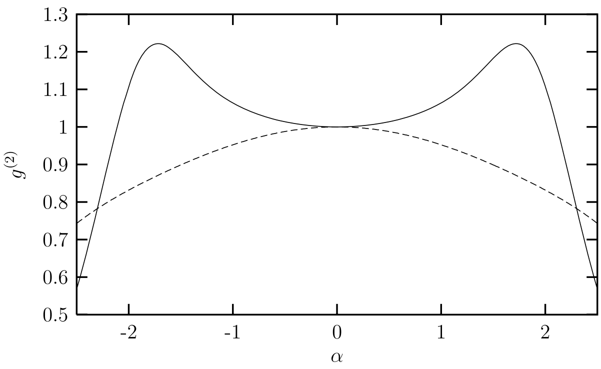

Fig. 1 Sub-Poissonian and super-Poissonian behaviour of the nonlinear coherent states constructed with the two different approaches discussed in the text. *Dotted line* $g_A^{(2)}$, *solid line* $g_D^{(2)}$

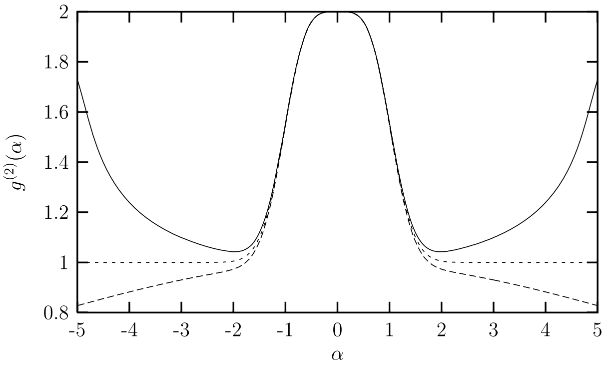

Fig. 2 Second order correlation function $g^{(2)}$ for the even combination of coherent states $|\alpha\rangle^{(+)}$ (*dotted line*), and the two nonlinear coherent states discussed in the text ($|\alpha\rangle_A^{(+)}$ *broken line*, $|\alpha\rangle_D^{(+)}$ *solid line*)

for the rest of the interval whereas the states $|\alpha\rangle_D$, constructed by the application of the approximate displacement operator (full line) show Poissonian behaviour when $\alpha \simeq 0$, super-Poissonian (bunching) for $|\alpha| \leq \alpha_0$ and sub-Poissonian (antibunching) for $|\alpha| \geq \alpha_0$. These results were obtained for a system with $N = 10$ and deformation function given by (3). Notice that the conduct of these states depends upon the kind of generalization used. As we increase the number of bound states the deformation function approaches unity and the behaviour of all these states converges to the harmonic result.

Schrödinger's cat has been the subject of an enormous number of papers and books [25] and physical realizations of a Schrödinger's cat have been achieved for example by the manipulation of photons in a cavity by dispersive atom-field coupling [26] and by the manipulation of the state of a single atom [27]. In these realizations a system with nearly macroscopic dimensions is excited into a superposition of two coherent states. Combinations of two coherent states of the harmonic oscillator algebra, result in non classical features such as sub-Poissonian and oscillatory number state distributions, squeezing, and interference [11, 12, 24].

We made even $|\alpha\rangle^{(+)}$, and odd $|\alpha\rangle^{(-)}$ combinations of coherent ($|\alpha\rangle$), and nonlinear coherent states ($|\alpha\rangle_A,\ |\alpha\rangle_D$) obtained using the methods discussed in this work. In Fig. 2 we show the second order correlation function $g^{(2)}$ with the averages taken between even combinations defined as

$$|\alpha\rangle_i^{(+)} = N_i^{(+)}(|\alpha\rangle_i + |-\alpha\rangle_i)$$

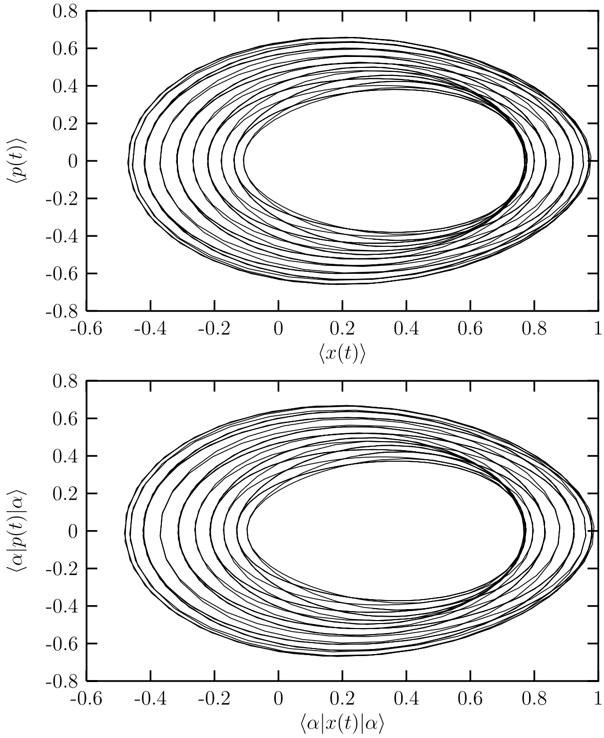

Fig. 3 Phase space trajectories for a system supporting $N = 10$ bound states with the averages made between nonlinear coherent states defined as eigenstates of the annihilation operator (*upper panel*) and as those obtained from a deformed displacement operator (*lower panel*). The value $|\alpha| = .5$ was used

with $N_i^{(+)}$ a normalization constant and the states $|\alpha\rangle_i$ coherent ($|\alpha\rangle$) and nonlinear $|\alpha\rangle_D$ or $|\alpha\rangle_A$. For the evaluation of the nonlinear coherent states we considered a system supporting $N = 30$ bound states so that the energies corresponding to the $|\alpha|$ values taken into account here belong to the lower part of the spectrum. For the combination of coherent states $g^{(2)} = 1$ for $|\alpha| \geq 2$ and increases up to 2 when $|\alpha| \simeq 0.5$. For the combination of nonlinear coherent states $|\alpha\rangle_D^{(+)}$ (broken line) $g_D^{(2)} < 1$ for $|\alpha| \geq 1.8$ and the behaviour for $|\alpha| \leq 1.2$ is similar to the harmonic case, finally, for $|\alpha\rangle_A^{(+)}$ (full line) $g_A^{(2)}$ behaves, for $|\alpha| \leq 1.2$ similarly to the other two cases but for $|\alpha| \geq 1.2$ it deviates from them and increases, it never attains the value of one.

The odd combination of coherent states is such that $g^{(2)}$ approaches one for $|\alpha| \geq 2$ as was the case for the even combination, however for smaller values of $|\alpha|$ it decreases smoothly attaining zero for $|\alpha| \simeq 0.4$. For the combinations of nonlinear coherent states $|\alpha\rangle_D^{(-)}$, and $|\alpha\rangle_A^{(-)}$ the second order correlation function has a similar behaviour as that of the harmonic case, however they deviate from each other as $|\alpha|$ increases. The second order correlation function obtained with $|\alpha\rangle_A^{(-)}$ takes values larger than one for $|\alpha| \geq 1.5$ while for case of $|\alpha\rangle_D^{(-)}$ it starts to decrease and never attains one being always bellow the harmonic values.

Figure 3 shows the phase space trajectories $\langle x(t)\rangle$ versus $\langle p(t)\rangle$ for a system with $N = 10$ and $\alpha = .5$ evaluated with approximate coherent states obtained as eigenstates of the deformed annihilation operator (upper panel) and as displaced vacuum (lower panel). We can see that for this value of the parameter α, both nonlinear coherent states yield a similar average value for the position and momentum. If we increase the value of $|\alpha|$ or diminish the number of bound states (thus increasing the anharmonicity) the differences will be more relevant.

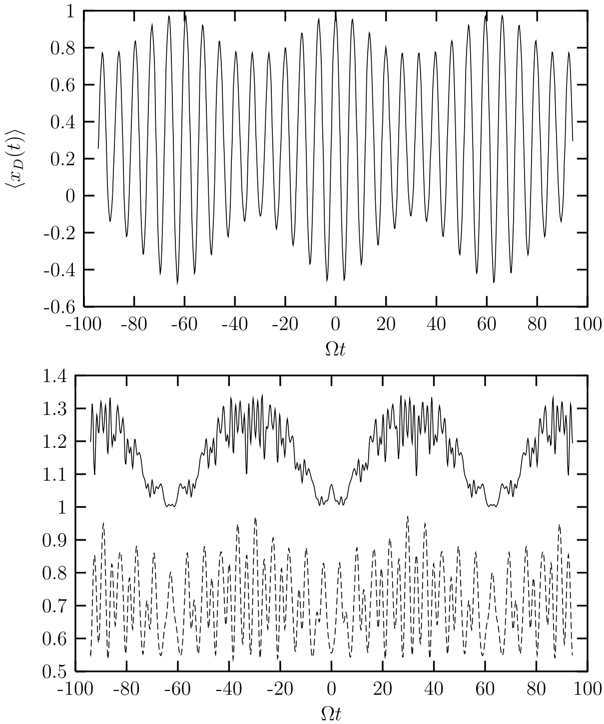

Fig. 4 Temporal evolution of the expectation value of the coordinate $\langle x_D\rangle$ corresponding to a nonlinear coherent state $|\alpha\rangle_D$ with $\alpha = 0.5$ of a deformed oscillator supporting $N = 10$ bound states (*upper panel*). In the *lower panel* we show the corresponding normalized dispersion $2\Delta x_D \Delta p_D/|\langle[x_D, p_D]\rangle|$ (*full line*) and the dispersion in the momentum Δp_D (*broken line*)

We also evaluated the dispersions in the deformed coordinate and momentum between nonlinear coherent states obtained with the displacement operator $D_D(\alpha)$. In [22] we evaluated them between Morse-like approximate coherent states obtained as eigenstates of the deformed annihilation operator and found that these approximate coherent states were not minimum uncertainty states though they showed a periodic behaviour with an amplitude near that of a minimum uncertainty state.

Figure 4 shows the temporal evolution of the average value of the deformed coordinate ${}_D\langle\alpha|x_D(t)|\alpha\rangle_D$ (upper panel) and in the lower panel we show the corresponding normalized dispersion $2\Delta x_D \Delta p_D/|\langle[x_D, p_D]\rangle|$ and the dispersion in the momentum Δp_D for fixed $\alpha = 0.5$ and a system supporting $N = 10$ bound states.

Notice that though the average energy is constant the amplitude of the oscillations is not (upper panel Fig. 4), and the uncertainty (full line lower panel Fig. 4) takes its largest values at those times when the amplitude of the oscillation is minimum. When the amplitude is largest the uncertainty is almost that of a minimum uncertainty state corresponding to a usual coherent state. With respect to the uncertainty in the momentum (dotted line, lower panel Fig. 4) we see that it takes values smaller than $1/\sqrt{2}$ indicating the presence of squeezing. These results are consistent with those obtained in [22] for the same set of parameters.

6 Conclusions

In this work we have presented an alternative proposal to generalize the displacement operator method for the construction of nonlinear approximate coherent states. The operator $D_D(\alpha)$ introduced in this work is nearly unitary and displaces both A, and $A^\dagger$. The nonlinear approximately coherent states thus constructed present a similar conduct as that of the approximate coherent states defined as eigenstates of an annihilation operator [22] in regards to the average values of position, momentum, and the dispersion relations. However they show different behaviour with respect to the statistics.

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